Solución Pep1 2008

1) $x^2-2x+3>0\ \forall x\in\mathbb{R}(pues\ D<0)\Leftrightarrow 5(x^2-2x+3)>|(x-3)(x-2)|$

a) $Si\ x<0\Rightarrow x-2<0;x-3<0\ \therefore 5x^2-10x+15>x^2-5x+6\Rightarrow 4x^2-5x+9>0$

$$\forall x\ (pues\ D<0)\therefore Sa=(-\infty-2)$$

b) $Si\ 2<x<3\Rightarrow\ x-2>0;x-3<0\therefore 5x^2-10x+15>-x^2+5x-6\Rightarrow 6x^2-15x+21>0\ \forall x\ (pues\ D<0)\therefore S_b=(-2,-3)$

c) $Si\ x>3:\ (x-2)(x-3)>0\Rightarrow 4x^2-5x+9>0\ \forall x(D<0)\therefore$

$$s_t=(-\infty,2)\ U(2,3)U(3,\infty):\ s_t=\mathbb{R}-\{2;3\}$$

2) $cos^2\left(\frac{\pi}{4}-\ \alpha\right)-sen^2\left(\frac{\pi}{4}-\ \alpha\right)=\ cos2\left(\frac{\pi}{4}-\alpha\right)=cos\frac{\pi}{2}\ cos2\alpha+sen\frac{\pi}{2}\ sen2\alpha=sen2\alpha$

3) $(foh)(x)=\frac{5h(x)}{3h(x)-1}=\frac{x}{x+3}\Leftrightarrow 5h(x)(x+3)=x(3h(x)+1\ )\Rightarrow\quad h(x)(2x+15)=x\therefore$
$h(x)=\frac{x}{2x+15}$

4) $\lim_{x\to 0}\frac{\frac{1}{2^{1/x}}+1}{\frac{3}{2^{1/x}}+1}=1\quad \frac{2}{3}\ si\ x\to 0^{+}_{-};\ \lim_{x\to\infty}\frac{-x}{1+x^2}=\lim_{x\to\infty}\frac{-1}{\frac{1}{x}+x}=0$

5) $y'(x)=\frac{\sqrt{1-a^2x^2}}{4a}-\frac{a^2x^2}{4a\sqrt{1-a^2x^2}}-\frac{1}{4a\sqrt{1-a^2x^2}}+xArcSen\ ax+\frac{ax^2}{2\sqrt{1-a^2x^2}}$

$$y'(x)=\frac{(1-a^2x^2)-a^2x^2}{4a\sqrt{1-a^2x^2}}+\frac{2a^2x^2}{4a\sqrt{1-a^2x^2}}+xArcSen\ ax-\frac{1}{4a\sqrt{1-a^2x^2}}=$$

$y'(x)=\ xArcSen\ ax$

6) a) $\log y=Arctg\ x+\log 10/(\,)'=\ \frac{y'}{y}=\frac{\log 10}{1+x^2}\Rightarrow y'=10^{Arctg\ x}\left(\frac{\log 10}{1+x^2}\right)$

b) $y'=n\left(x+\sqrt{x^2+a^2}\right)^{n-1}\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)=\frac{n\left(x+\sqrt{x^2+a^2}\right)}{\sqrt{x^2+a^2}}=\frac{ny}{\sqrt{x^2+a^2}}$

$$y''=\frac{ny'\sqrt{x^2+a^2}-\frac{nxy}{\sqrt{x^2+a^2}}}{x^2+a^2}=\frac{n[(x^2+a^2)y'-xy]}{(x^2+a^2)^{3/2}}$$

$$(x^2+a^2)y''=\frac{n(x^2+a^2)\frac{ny}{\sqrt{x^2+a^2}}-nxy}{(x^2+a^2)^{1/2}}=n^2y-\frac{nxy}{\sqrt{x^2+a^2}}$$

$$-xy'+n^2y=\frac{-nxy}{\sqrt{x^2+a^2}}+n^2y\qquad\therefore (x^2+a^2)y''=-xy'+n^2y$$