Solución Pep 2 2007

1.-

$y'(x) = \lim_{h\to 0} \frac{1}{h}(2\cos 2(x+h) - 2\cos 2x) = \lim_{h\to 0} \frac{2}{h}(\cos 2x \cos 2h - sen\ 2x\ sen\ 2h - \cos 2x)$

$$y'(x) = \lim_{h\to 0} \frac{2\cos 2x(\cos 2h - 1)}{h} - \lim 2\text{sen } 2\text{x}\left(\frac{\text{sen } 2\text{h}}{2\text{h}} 2\right)$$

$$y'(x) = \lim \frac{2\cos 2x(cos}{}$$

2.-a) $y'(x) = \frac{\sqrt{x^2-1}\frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}} - Ln\left(x+\sqrt{x^2-1}\right)\frac{x}{\sqrt{x^2-1}}}{x^2-1} = \frac{\sqrt{x^2-1} - xLn\left(x+\sqrt{x^2-1}\right)}{(x^2-1)^{3/2}}$

$$(1-x^2)y' = -\frac{\sqrt{x^2-1} - xLn(x+\sqrt{x^2-1})}{(x^2-1)^{1/2}}$$

$$-xy = -\frac{xLn\left(x+\sqrt{x^2-1}\right)}{(x^2-1)^{1/2}} \Rightarrow (1-x^2)y' - xy + 1 = 0$$

b) $y' = \frac{\left(\frac{1}{-\sqrt{1-x^2}}\right)\sqrt{1-x^2} + \frac{x\ arc\cos x}{\sqrt{1-x^2}}}{(1-x^2)} = \frac{-\sqrt{1-x^2} + x\ arc\cos x}{(1-x^2)^{3/2}}$

$$(1-x^2)y' = \frac{-\sqrt{1-x^2} + x\ arc\cos x}{(1-x^2)^{1/2}}$$

$$-xy = -\frac{x\ arc\cos x}{(1-x^2)^{1/2}} \Rightarrow (1-x^2)y' - xy + 1 = 0$$

3.- a) $(f\circ g)(x) = f\left(g(x)\right) = \frac{1}{g^2(x)} = \frac{2x^2-6x+1}{x} = 2x - 6 + \frac{1}{x}$

$$(f\circ g)'(x) = 2 - \frac{1}{x^2}$$

b) $f'^{\left(g(x)\right)} = f'(u)g'(x) = -\frac{2}{u^3}\frac{\frac{1}{2\sqrt{x}}\sqrt{2x^2-6x+1} - \sqrt{x}\frac{4x-6}{2\sqrt{2x^2-6x+1}}}{(2x^2-6x+1)}$

$$f'\left(g(x)\right) = -\frac{1}{u^3}\frac{\frac{1}{\sqrt{x}}2x^2 - 6x + 1 - \sqrt{x}\,(4x-6)}{(2x^2-6x+1)^{3/2}} = -\frac{1}{u^3}\frac{(2x^2-6x+1) - 4x^2 + 6x}{\sqrt{x}(2x^2-6x+1)^{3/2}}$$

$$f'\left(g(x)\right) = -\frac{1}{u^3}\frac{1-2x^2}{\sqrt{x}(2x^2-6x+1)^{3/2}} = -\frac{(2x^2-6x+1)^{3/2}}{x^{3/2}}\frac{(1-2x^2)}{\sqrt{x}(2x^2-6x+1)^{3/2}}$$
$$= -\frac{1}{x^2} + 2$$

4.- $y' = -2^2 \sin 2x$; $y'' = -2^3 \cos 2x$;

$y''' = 2^4 \sin 2x$; $y^{iv} = 2^5 \cos 2x$;

$\therefore$ $y^{2n-1} = (-1)^n 2^{2n} \sin 2x$; $y^{2n} = (-1)^n 2^{2n+1} \cos 2x$;

5.- Derivando implícitamente: $\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = -\frac{b^2 x}{a^2 y}; pendiente \;\therefore$

$$(y - y_0) = \left(-\frac{b^2 x}{a^2 y}\right)(x - x_0) \Rightarrow xx_0 b^2 + yy_0 a^2 = a^2 {y_0}^2 + b^2 {x_0}^2 \Big/ \frac{1}{a^2 b^2} \Rightarrow$$

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = \frac{x_0}{a^2} + \frac{y_0}{b^2} = 1$$