UNIVERSIDAD DE SANTIAGO DE CHILE
Departamento de Matemática y CC.
Coordinación de Calculo Aplicado.-

Control # 6.- 29/06/2007.-

Nombre.______________________________Prof.______________

1.- Usando el método adecuado calcule las primitivas:

a) $\int \frac{1}{\sqrt{3-2x-x^2}}dx$ b) $\int \frac{dx}{x^3-1}$ c) $\int \frac{Sen2x}{Senx}dx$

d) $\int x^2 e^{2x^3}dx$ e) $\int Cos^7 xdx$ f) $\int \frac{dx}{1+\sqrt{x}}$.

---

a) $\displaystyle\int \frac{1}{4-(x+1)^2}\,dx =$ Sustitución trigonométrica.

$x+1 = 2\,\mathrm{sen}\,t$

$dx = 2\cos t \, dt.$

$\displaystyle\int dt = t + c$

$t = \mathrm{Arc\,sen}\,\frac{x+1}{2}$

$$\int \frac{2\cos t\, dt}{\sqrt{4-4\,\mathrm{sen}^2 t}} = \mathrm{Arc\,Sen}\,\frac{(x+1)}{2} + C$$

b) $\displaystyle\int \frac{dx}{(x-1)(x^2+x+1)}$ FRACCIONES parciales

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+c} \Rightarrow 1 = A(x^2+x+1) + (Bx+C)(x+1)$$

$A+B=0$ | $A = 1/3$

$A-B+C=0$ | $B=-1/3$

$A-C=1$ | $C=-2/3$

$$I = \frac{1}{3}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{x+2}{x^2+x+1}\,dx = \frac{1}{3}\ln(x-1) - \frac{1}{3}\int\frac{x+1}{x^2+x-1}$$

$$I = \ln(x-1)^{1/3} - \frac{1}{6}\int\frac{2x+1+1}{x^2+x+1}\,dx = \ln(x-1)^{1/3} - \frac{1}{6} I_2$$

$$I_2 = \frac{1}{6}\int\frac{2x+1}{x^2+x+1}\,dx + \frac{1}{2}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{1}{6}\ln(x^2+x+1) + \frac{1}{6}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$I = \ln(x-1)^{1/3} - \frac{1}{6}\ln(x^2+x+1) + \frac{1}{2}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$I = \ln\frac{(x-1)^{1/3}}{(x^2+x+1)^{1/6}} + \frac{1}{2}\,\mathrm{Arctg}\,\frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\cdot\frac{2}{\sqrt{3}}$$

$$I = \ln\frac{(x-1)^{1/3}}{(x^2+x+1)^{1/6}} + \frac{1}{\sqrt{3}}\,\mathrm{Arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + C_1$$

c) $\int \frac{2 \operatorname{sen} x \cos x}{\operatorname{sen} x} = 2\int \cos x \, dx = 2 \operatorname{sen} x + C$

d) $\int x^2 e^{2x^3} dx =$ $\quad u = 2x^3 \quad du = 6x^2 dx$

$\frac{1}{6}\int e^u du = \frac{1}{6} e^u + c = \frac{1}{6} e^{2x^3} + C$

e) $\int \cos^6 x \cdot \cos x \, dx = \int (1 - \operatorname{sen}^2 x)^3 \cos x \, dx$

Sustitucion.

$u = \operatorname{sen} x$

$u^2 = \operatorname{sen}^2 x$

$2u\,du = 2 \operatorname{sen} x \cos x \, dx$

$\cancel{u}\, du = \cancel{u} \cos x \, dx$

$du = \cos x \, dx$

$= \int (1 - u^2)^3 du$

$= \int (1 - 3u^2 + 3u^4 - u^6) du = u + u^3 + \frac{3u^5}{5} - \frac{u^7}{7}$

$= \operatorname{Sen} x + \operatorname{Sen}^3 x + \frac{3}{5} \operatorname{sen}^5 x + \frac{1}{7} \operatorname{sen}^7 x$

f) $\int \frac{dx}{1+\sqrt{x}}$ Sustitucion $\sqrt{x} = u^2$ o $x = u^2$

$\int \frac{2u\,du}{1+u} = 2\int \frac{u+1-1}{1+u} du = 2\int du - 2\int \frac{du}{1+u} = 2u - 2\ln(1+u)$

$I = 2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$