Solución pep2 2006

1.- a) $g\left(h(x)\right)=\frac{h-2}{h+5}=\frac{2x}{x+1}\Rightarrow(h-2)(x+1)=2x(h+5)\Rightarrow h(1-x)=12x+2$;

$h(x)=\dfrac{12x+2}{1-x}$ $\qquad$ $Si\ y=\dfrac{12x+2}{1-x}\Rightarrow x(12+y)=y-2\Rightarrow x=\dfrac{y-2}{y+12}$

$\therefore h^{-1}(y)=\dfrac{y-2}{y+12}$

b) $y=(f\circ g)(x)=f\left(g(x)\right)=\frac{2g}{g+1}=\frac{2\left(\frac{x-2}{x+5}\right)}{\frac{x-2}{x+5}+1}=\frac{2(x-2)}{x-2+x+5}=\frac{2(x-2)}{2x+3}\Rightarrow$

$y=\dfrac{x-2}{x+5}\Rightarrow x(y-1)=-2-5y\Rightarrow x=\dfrac{5y+2}{1-y}\Rightarrow g^{-1}(x)=\dfrac{5x+2}{1-x}$

$y=\dfrac{2x}{x+1}\Rightarrow x(y-2)=-y\Rightarrow x=\dfrac{y}{2-y}\Rightarrow f^{-1}(x)=\dfrac{x}{2-x}$

$g^{-1}\circ f^{-1}=\left(\dfrac{5f^{-1}+2}{1-f^{-1}}\right)=\dfrac{5\frac{x}{2-x}+2}{1-\frac{x}{2-x}}=\dfrac{5x-2x+4}{2-2x}=\dfrac{3x+4}{2(1-x)}$

2.- a) i) $2cot^2\alpha+cot^4\alpha=\frac{2\cos^2\alpha}{\sin^2\alpha}+\frac{\cos^4\alpha}{\sin^4\alpha}\equiv\frac{2\sin^2\alpha\cos^2\alpha+\cos^4\alpha}{\sin^4\alpha}=\frac{\cos^2\alpha+\sin^2\alpha\cos^2\alpha}{\sin^4\alpha}$

ii) $csc^4\alpha-1=\frac{1-\sin^4\alpha}{\sin^4\alpha}=\frac{(1-\sin^2\alpha)(1+\sin^2\alpha)}{\sin^4\alpha}=\frac{\cos^2\alpha(1+\sin^2\alpha)}{\sin^4\alpha}$

b) $tan^2x-\tan x+3\tan x-3=2\tan x\Leftrightarrow tan^2x=3\ \therefore\tan x=\pm\sqrt{3}\Rightarrow x=\pm\frac{\pi}{3}+k\pi$

3.- a) $Si\ y=\frac{-5}{12}x+5\Rightarrow m=-\frac{5}{12}\Rightarrow m=\frac{12}{5}\ pendiente\ de\ recta\ pedida$

$\therefore y-12=\dfrac{12}{5}(x-17)\Leftrightarrow 5\left(5-\dfrac{5}{12}x\right)-60=12(x-17)\Leftrightarrow$

$\left(12+\dfrac{25}{12}\right)x=-35+17*12\Rightarrow x=\dfrac{169}{\frac{169}{12}}\Rightarrow x=12;y=0$

c) $d=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}=\frac{|5x_0+12y_0-60|}{\sqrt{25+144}}=\frac{1}{13}|5\cdot17+12\cdot12-60|=\frac{169}{13}=13$

4.- a) $(x-h)^2=4p(y-k)\ la\ parábola\Rightarrow 2\left(x^2-\frac{3}{2}x\right)=y-b\Rightarrow x^2-\frac{3}{2}x=\frac{y}{2}-3$

$\therefore\left(x-\dfrac{3}{4}\right)^2=\dfrac{y}{2}-3+\dfrac{9}{16}=\dfrac{y}{2}+\dfrac{39}{16}\Rightarrow\left(x-\dfrac{3}{4}\right)^2=4\dfrac{1}{8}\left(y+\dfrac{78}{16}\right)\therefore p={}^1\!/_8$

$V\left(\dfrac{3}{4},-\dfrac{39}{8}\right):F\left(\dfrac{3}{4},\dfrac{1}{8}-\dfrac{39}{8}\right)=\left(\dfrac{3}{4},-\dfrac{19}{4}\right)$

b) Si $y = 5x + b$ $intersección\ con\ y = 2x^2 - 3x + 6$. $5x + b = 2x^2 - 3x + 6$

$$2x^2 - 8x + 6 - b = 0\ condición\ discriminante = 64 - 8(6 - b) = 0 \Rightarrow b = -2$$

$\therefore y = 5x - 2\ la\ tangente$

$la\ normal\ tiene\ pendiente\ m = -{}^{1}/_{5}\ y\ pasa\ por\ la\ intersección$

$\Rightarrow 2x^2 - 8x + 8 = 0 \Rightarrow x = 2 \wedge y = 8$ $\therefore (y - 8) = -\frac{1}{5}(x - 2)\ \ la\ normal$