**UNIVERSIDAD DE SANTIAGO DE CHILE**
**DEPARTAMENTO DE MATEMÁTICA Y C.C**
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# CALCULO APLICADO
## PRUEBA Nº 3
## (Solución)

**1.**

$$L_C = \int_0^3 \sqrt{1 + y`^2(x)}\,dx$$

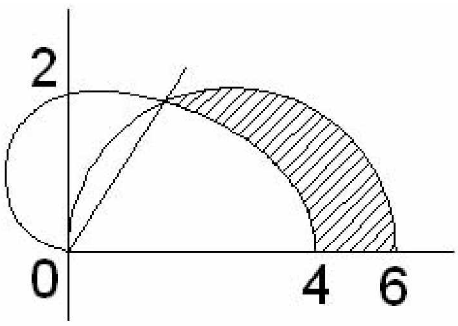
$$y` = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$$

$$1 + y`^2 = \frac{1}{4}\left(\frac{1}{x} + x + 2\right) = \frac{1}{4}\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)^2$$

$$L_C = \frac{1}{2}\int_0^3 \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)dx = \frac{1}{2}\left(2\sqrt{x} + \frac{2}{3}x^{3/2}\right)\Bigg|_0^3$$

$$\therefore L_C = \frac{1}{2}\left(2\sqrt{3} + \frac{2}{3}\sqrt{3^3}\right)$$

**2.**




$$6\cos\theta = 2(1+\cos\theta) \Rightarrow 4\cos\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$A = \frac{1}{2}\cdot 2\int_0^{\frac{\pi}{3}} \left(\rho_1^2 - \rho_2^2\right)d\theta$$

$$A = \int_0^{\frac{\pi}{3}} \left(36\cos^2\theta - 4(1+\cos\theta)^2\right)d\theta$$

$$A=\int_0^{\frac{\pi}{3}}\left(-8\cos\theta-4+32\cos^2\theta\right)d\theta=\left(-8sen\theta-4\theta+16(\theta+sen\theta\cos\theta)\right)\Big|_0^{\pi/3}$$

$$\therefore A=-8\frac{\sqrt{3}}{2}+4\pi+16\frac{\sqrt{3}}{4}=4\pi$$

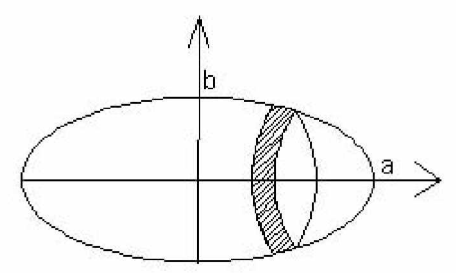
**3.**

$$A(s)=2\pi\int_0^3 x\sqrt{1+y'^2(x)}\,dx$$

$$A(s)=2\pi\int_0^3 x\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)dx=2\pi\left(\frac{2}{3}x^{3/2}+\frac{2}{5}x^{5/2}\right)\Bigg|_0^3$$

$$\therefore A=2\pi\left(\frac{2}{3}\cdot 3^{3/2}-\frac{2}{5}\cdot 3^{5/2}\right)$$

**4.**




$$V=2\pi\int_0^a b^2\left(1-\frac{x^2}{a^2}\right)dx$$

$$V=2\pi\left(b^2x-\frac{b^2}{a^2}\cdot\frac{x^3}{3}\right)\Bigg|_0^a$$

$$\therefore V=\frac{2ab^2}{3}$$

**5.**

$$A=\int_0^{2\pi} y(t)x`(t)dt$$

$$A=3^2\int_0^{2\pi}(1-\cos t)(1-\cos t)dt=9\int_0^{2\pi}\left(1-2\cos t+\cos^2 t\right)dt$$

$$A=9\left(t-2sent+\frac{1}{2}t+sent\cos t\right)\Big|_0^{2\pi}$$

$$\therefore A=27\pi$$