UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
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CALCULO APLICADO
PRUEBA Nº 4
(Solución)

1.

a) Como $\frac{\partial g}{\partial x} = 4 \wedge \frac{\partial f}{\partial y} = -2; \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \neq 0 \Rightarrow F$ No es campo gradiente.

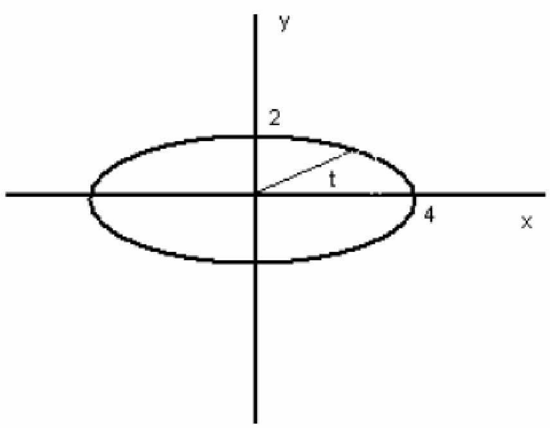
$$\oint_C [(x^2 - 2y)dx + (4x - y^3)dy] : C : \begin{cases} x = 4\cos t \\ y = 2sent \end{cases}$$

b)

$$(i) \oint_C = \int_0^{2\pi} (-64 \cdot \cos^2(t) \cdot sen(t) - 16 sen^3(t) \cdot \cos(t) + 16 + 16\cos^2(t)] dt$$

$$= 16 \int_0^{2\pi} (1 + \cos^2(t))dt = 48\pi$$

$$(ii) \iint_D (4+2) dA = 6A(D) = 48\pi$$



2.

Hay dos opciones:

a) Parametrizar la recta que une P y Q.

b) Determinar un potencial pues $\nabla \times F = 0$, o sea F es campo gradiente; o sea $\oint_c$ es independiente del camino.

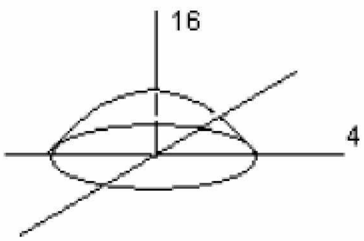
$$\phi(P) = xyz \Rightarrow w = \int F(P) dP = \phi(a,b,c) - \phi(1,1,1) = abc - 1$$

3.

$$A(S)=\iint_D \sqrt{1+{Z_x}^2+{Z_y}^2}\,dA_{xy}=\iint \sqrt{1+4(x^2+y^2)}\,dA_{xy}$$ Pero en coordenadas polares:

$$A(S)=4\int_0^{\pi/2}\int_0^4 \sqrt{1+4\rho^2}\,\rho\,d\rho\,d\theta=4\int_0^{\pi/2}(\frac{1}{8}(1+4\rho^2)\cdot\frac{2}{3}d\theta$$

$$A(S)=\frac{\pi}{6}\left(65^{3/2}-1\right)$$



4.

Con coordenadas esféricas:

$$V=4\int_0^{\pi/2}\int_0^{\pi/4}\int_2^4 \rho^2 sen\varphi\,d\rho\,d\varphi\,d\theta$$

$$V=4\int_0^{\pi/2}\int_0^{\pi/4}(\frac{4^3-2^3}{3})sen\varphi\,d\varphi\,d\theta$$

$$V=4\int(\frac{4^3-2^3}{3})(-\cos\varphi\ \Big|_0^{\pi/4}d\theta$$

$$V=4\cdot(1-\frac{\sqrt{2}}{2})\cdot(\frac{4^3-2^3}{3})\cdot\frac{\pi}{2}$$