UNIVERSIDAD DE SANTIAGO DE CHILE
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CALCULO APLICADO
PRUEBA Nº 3
(Solución)

1.

$$\lim_{\substack{x\to 0\\ y\to 0}}\left(\frac{y\cdot sen(xy)}{1-e^{x^2+y^2}}\right)=\lim_{\substack{x\to 0\\ y\to 0}}\frac{sen(xy)}{xy}\cdot\frac{x^2+y^2}{1-e^{x^2+y^2}}\cdot\frac{xy^2}{x^2+y^2}=\lim_{\substack{x\to 0\\ y\to 0}}\left(\frac{xy^2}{x^2+y^2}\right)=$$

$$=\lim_{\substack{\rho\to 0\\ \theta\to\theta}}\left(\frac{\rho^3\cdot\cos\theta\cdot sen^3\theta}{\rho^2}\right)=0$$

Si f(0,0)=0; es continua

2.

$$a)\,f(x,y)=\frac{xy^3-x^3y}{x^2+y^2};\frac{\partial f}{\partial y}=\frac{(x^2+y^2)(x^3-3xy^2)-2y(x^3y-xy^3)}{(x^2+y^2)}$$

$$\frac{\partial f}{\partial y}(0,0)=\lim_{k\to 0}\frac{f(0,k)-f(0,0)}{k}=0\wedge\lim_{\substack{x\to 0\\ y\to 0}}\frac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}=$$

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{\rho^5(\cos^5(\theta)-4\cos^3(\theta)\cdot sen^2(\theta)-\cos(\theta)\cdot sen^4(\theta)}{\rho^4}=0\Rightarrow\frac{\partial f}{\partial y}\_es\_continua\_en\_(0,0)$$

$$b)\frac{\partial f}{\partial y}(x,0)=\frac{x^5}{x^4}x\Rightarrow\frac{\partial^2 f}{\partial x\partial y}(x,0)=1\wedge\frac{\partial^2 f}{\partial x\partial y}(0,0)=1$$

3.

$$z=f(t);t=\frac{x+y}{x\cdot y};\frac{\partial z}{\partial x}=f\,'(t)\cdot\frac{xy-y(x+y)}{x^2y^2}=-\frac{f\,'}{x^2}$$

$$\frac{\partial z}{\partial y}=\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots=-\frac{f\,'}{y^2}\therefore$$

$$x^2\frac{\partial z}{\partial x}-y^2\frac{\partial z}{\partial y}=0$$

4.

$$a)\begin{matrix} f_x = 2x-2=0 \\ f_y = 2y+4=0 \end{matrix} \Rightarrow P_o(1,-2); f_{xx}=2; f_{yy}=2 : f_{xy}=0$$

$$H(1,-2)>0 \wedge f_{xx}>0 \Rightarrow P_o ...es..un..punto..\min imo$$

$$a) f(x,y)=(x-1)^2+(y-2)^2+1 \geq 1 \Rightarrow f(1,-2)=1 \Rightarrow \min imo$$

$$b)(x-0)f_x(P_o)+(y-0)f_y(P_o)=z-6 : \frac{\partial f}{\partial x}(P_o)=-2 : \frac{\partial f}{\partial y}(P_o)=4 \therefore$$

$$-2x+4y=z-6 \vee 2x-4y+z=6$$

$c)$ Recta_Normal :

$$\frac{x}{-2}=\frac{y}{4}=\frac{z-6}{-1}$$

si

$z=0$

$x=-12; y=24 \therefore en\_(-12,24)$

$$z-1=(x-1)^2+(y-2)^2$$